

Coupled SIS Models

The epidemiological models that we have seen so far have involved a homogeneous, well-mixed population. We can also develop a model that incorporates the interaction between two distinct, homogeneous, well-mixed populations. An example of where this type of model might be a isolated population (say on an island) that is visited by a different population. In this project, you will develop a coupled SIS model for each population. If a disease is endemic in one population but not in the other, under what conditions would the coupling cause the non-endemic population to become endemic?

Some Model Requirements

- The model without coupling is a SIS-type model, which is similar to an SIR-type model except that there is no immunity.
- An uncoupled SIS model divides the population into two groups: Susceptible (S) and Infected (I). There is no recovered group so recovered infected members become susceptible again.
- Susceptible members are infected at a rate proportional to their population size and the size of the Infected population.
- Infected members recover to become susceptible members at a rate proportional to the size of the infected population.
- A coupled SIS-type model consists of two independent SIS populations that are coupled together.
- In the coupled model, each population is divided into two groups of susceptible and infected members for a total of four groups (two for each population).
- The infected members from Population 1 (I_1) infect susceptible members of Population 2 (S_2) at a rate proportional to the size of the infected group in Population 1 and the size of the susceptible group in Population 2.
- The reverse coupling is also true so infected members of Population 2 infect susceptible members of Population 1.
- There are quite a few parameters in this system. Your model should include them all but can make various assumptions to simplify the analysis. Things like assuming the total size of each population is equal or assuming the different infection rates are the same would be reasonable assumptions.

Some Questions to Answer

- What are the dynamics of an single uncoupled SIS-type model? Under what conditions would the infection be endemic?
- Consider of two coupled SIS populations. If the infection is not endemic in one but endemic in another, under what conditions would the they both become endemic?
- Could the coupling cause the infection to cease to be endemic in both?
- How does the dynamics between uncoupled and coupled SIS-type systems? What are the steady states and their stability? What does the phase plane look like?